Journal of Visualization, Vol. 8, No. 4 (2005)323-330

Taylor-Görtler Vortices in the Flow Driven by a Rotating Magnetic Field in a Cylindrical Container

Fraňa, K.*, Stiller, J.* and Grundmann, R.*

* Institute for Aerospace Engineering, TU Dresden, D-01062 Dresden, Germany.
E-mail: frana@tfd.mw.tu-dresden.de

Received 2 May 2005
Revised 4 July 2005

Abstract : Taylor-Görtler vortices play an important role in the flow driven by rotating magnetic field in a closed cylindrical container. The visualization of these structures is essential to understand their physics and their impact on the dynamics of the flow. We present a study of various methods for identifying vortex cores, including the so-called λ_2 and Q criteria. While λ_2 is, probably, the better indicator it is more sensitive to numerical noise and thus less effective in devising 3D vortex structures. Here, the Q criterion proved particularly useful when applied to the fluctuation velocity field.

Keywords : Taylor-Görtler vortices, Visualization, Identification, Rotating flow.

1. Introduction

The incompressible flow driven by a rotating magnetic field (RMF) in a finite length cylindrical container plays an important role in single crystal growth and in metallurgical processes. From a practical point of view, the RMF generates a primary swirling flow in fluid. Furthermore, a weak meridional flow is driven by the Bödewadt type boundary layers which develop at the top and bottom walls. Figure 1(a) shows the mean velocity distribution. A snapshot of the instantaneous and corresponding fluctuation fields is given in Figs. 1(b) and 1(c), respectively. Although the impact of Taylor-Görtler type vortices on the flow is evident particularly in the fluctuation velocity field, an identification of these vortex structures is required for deeper understanding.

Numerical investigations by different authors revealed that pairs of Taylor-Görtler vortices are generated at the rim in the middle region of the cylinder if a critical Taylor number (ratio between magnetic and viscous forces) is exceeded. Due to the effect of secondary flow, these vortices are shifted toward the extremities of the cylinder where they are finally dissipated (Kaiser and Benz, 1998), (Grants and Gelfgat, 1998). Moreover, the existence of these vortices was confirmed experimentally by measurements of the azimuthal velocity component (Marty and Martin Witkowski, 1999). While previous numerical investigations assumed axisymmetric conditions, Grants and Gerbeth (2002) proved that the critical perturbations are three-dimensional, and thus promote the development of non-axisymmetric structures.

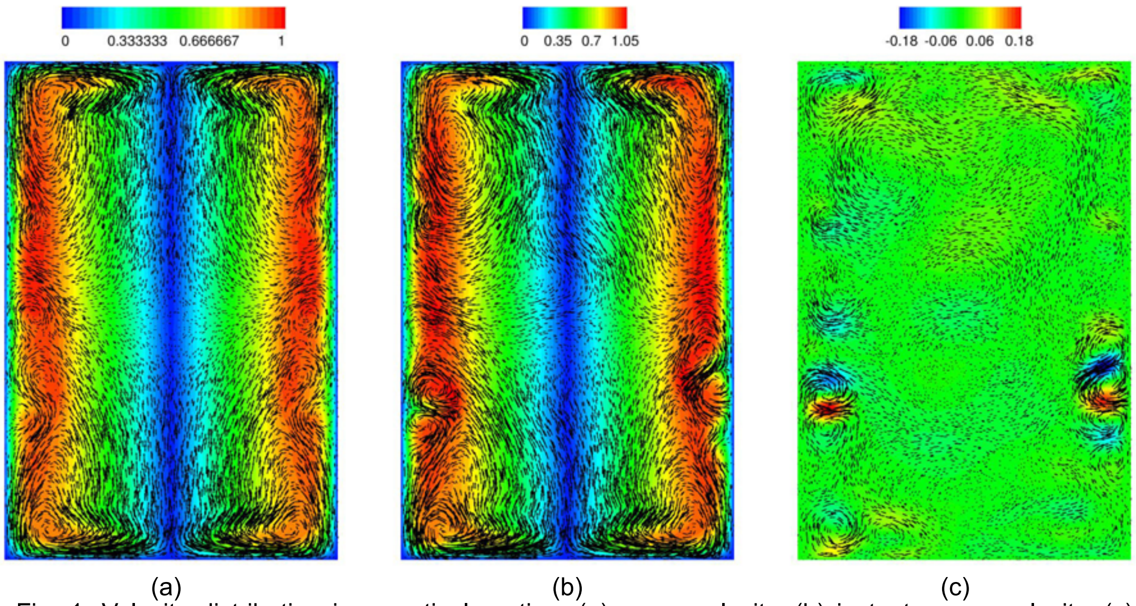


Fig. 1. Velocity distribution in a vertical section: (a) mean velocity, (b) instantaneous velocity, (c) fluctuating velocity. The velocities are scaled by the maximal value of the mean azimuthal velocity.

The motivation of this paper is to investigate the nature and dynamics of these vortex structures and to devise suitable techniques of their identification and visualization. For this purpose we employ a dataset obtained from Direct Numerical Simulation (DNS) of the flow in a finite-length cylinder with an aspect ratio of $H/D = 1.5$, where H denotes the height and D the diameter. A Taylor number of $Ta = 1.125\ Ta^{cr}$ was chosen, where $Ta^{cr} = 40{,}079$ denotes the critical Taylor number provided by Grants and Gerbeth (2002). In this parameter range, the flow is weakly turbulent and expected to be governed by large, long-lived structures. The DNS was performed using a finite element code based on a pressure stabilized Petrov-Galerkin method with linear shape functions (Stiller et al., 2004). The computational grid consisted of 1.8 million points and 10 million tetrahedral cells, which corresponds to a mesh spacing of $h = D/100$.

In the next section, several techniques for vortex identification are introduced. In Section 3, the efficiency of these methods is examined for a two-dimensional representation of the flow. In Section 4, this analysis is extended to the 3D case. Finally, the conclusions are given in Sec. 5.

2. Vortex Identification Methods

It is generally assumed that the dynamics of turbulent shear flows is dominated by spatially coherent vortical structures. A sound visualization of these structures is definitely of great importance for understanding turbulence physics. Yet, the definition of a vortex is a problem for its own. In practice, any useful definition is intimately connected with an algebraic criterion. A simple approach follows from the observation that the presence of a vortex is usually indicated by a local pressure minimum. Thus, the vortex core could be identified as the region where the pressure drops below a certain threshold. While this criterion provides acceptable results in many cases it tends to fail systematically in situations where unsteady straining or viscous effects are of significance (Jeong and Hussain, 1995). In view of this deficiency, several alternative approaches have been developed. Most of these are based on the velocity gradient $\nabla\mathbf{u}$ rather than the pressure p. Examples include the vorticity magnitude $\nabla \times \mathbf{u}$ (Metcalfe et al., 1985), the tensor invariants of $\nabla\mathbf{u}$ (Hunt, 1988), (Chong et al., 1990), and the eigenvalues of the symmetric tensor $\mathbf{S}^2 + \mathbf{\Omega}^2$ where $\mathbf{S}$ and $\mathbf{\Omega}$ are the symmetric and antisymmetric parts of $\nabla\mathbf{u}$, respectively (Jeong and Hussain, 1995). These and a few further criteria were extensively discussed by Jeong and Hussain (1995). Here, we focus on the so-called Q and λ_2 criteria which are among the most promising.

The eigenvalues, σ, of the velocity gradient $\nabla\mathbf{u}$ satisfy the characteristic equation

$$\sigma^3 - P\sigma^2 + Q\sigma - R = 0\,,$$

where

$$P = u_{i,i} \equiv \nabla\cdot\mathbf{u},\quad Q = \tfrac{1}{2}(u_{i,i}^2 - u_{i,j}u_{j,i}),\quad R = \det(\nabla\mathbf{u})$$

are the tensor invariants and $u_{i,j} = \partial u_i / \partial x_j$. Assuming incompressible flow, the first invariant, P, is vanishing. Chong et al. (1990) noted that vortical motions are associated with complex eigenvalues and, hence, a positive discriminant of the characteristic equation, i.e.,

$$D = (\tfrac{1}{3}Q)^3 + (\tfrac{1}{2}R)^2 > 0\,.$$

Hunt et al. (1988) defined a vortex as the region with positive Q and p lower than the ambient pressure. As $Q > 0$ implies that D is positive as well, the criterion will generally indicate the existence of closed or spiral streamlines in a reference frame moved with the given point. While the Q criterion was found to produce much more reliable results than pure pressure or vorticity criteria, some deficiencies have been identified by Jeong and Hussain (1995). In particular, the Q criterion may fail if the vortex is exposed to a non-uniform strain field. As a remedy the authors proposed the so-called λ_2 criterion. Here, λ_2 is median of the eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$ of the symmetric tensor $\mathbf{S}^2 + \mathbf{\Omega}^2$ where

$$S_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}),\quad \Omega_{ij} = \tfrac{1}{2}(u_{i,j} - u_{j,i})$$

are the symmetric and antisymmetric parts of $\nabla\mathbf{u}$, respectively. Jeong and Hussain (1995) defined a vortex as a connected region with two negative eigenvalues or, equivalently, $\lambda_2 < 0$. Based on a discussion of the vorticity transport equation the authors conjectured that their definition considers only vortical contributions to local pressure minima while discarding effects of unsteady straining and viscous diffusion. The authors also showed that the Q criterion can be expressed as

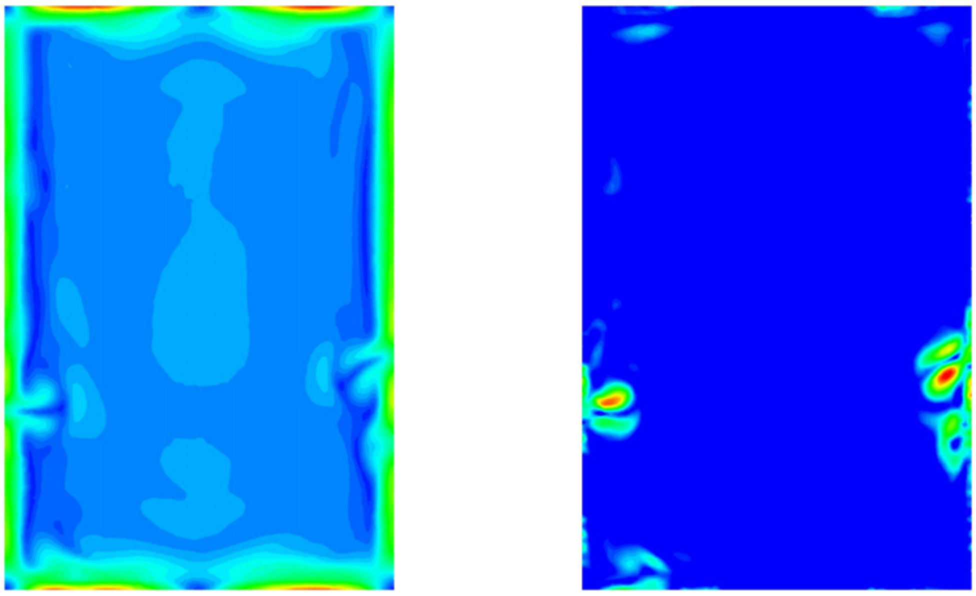
$$Q = -\tfrac{1}{2}(\lambda_1 + \lambda_2 + \lambda_3) > 0 \quad \text{(inside a vortex)}\,.$$

Obviously, both criteria are related but not equivalent. Although λ_2 is expected to be the more precise indicator, it is appropriate to consider Q as an alternative.

3. Two-Dimensional Vortex Visualization

To evaluate the efficiency of the criteria, the vortex identification is first performed in a meridional section of the flow (cf. Fig. 1). Since the main interest is in transitional and turbulent structures, the criteria are applied to the instantaneous, as well as to the fluctuating velocity fields. The primary objective is to identify the most appropriate method providing an unambiguous identification and clear insight into the TG vortex structures.

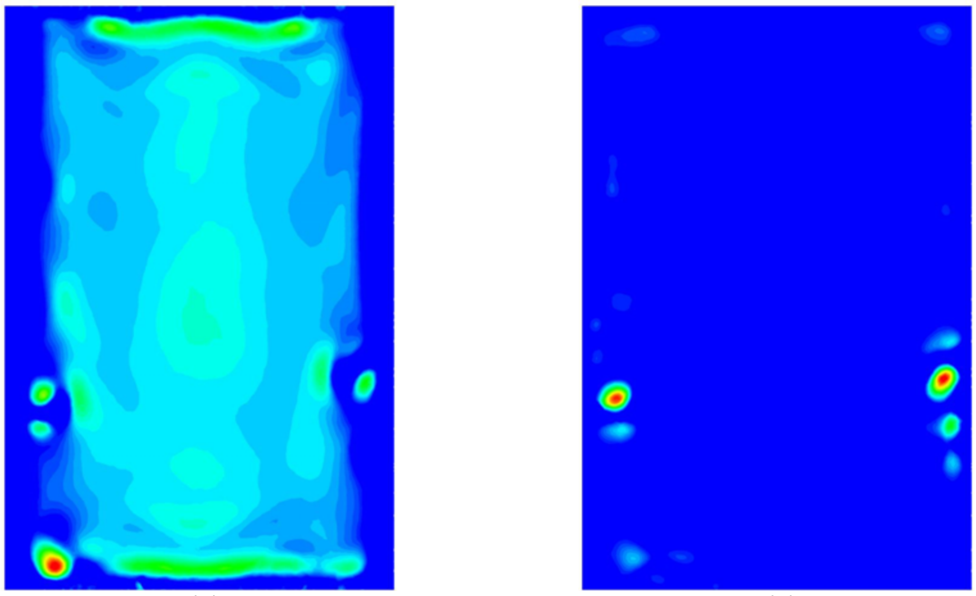
Figure 2(a) depicts the vorticity magnitude computed from the instantaneous velocity field. At the first glance, the vortices are identified but strongly interfered by the shear layers near the walls. Figure 2(b) shows the vorticity magnitude of the fluctuation velocity field. Obviously, the vortex cores are well identified while effects of mean shear flow disappeared completely. However, the contribution of instantaneous straining is still significant. Due to the strong presence of those strain effects, the vorticity magnitude is not suitable for a clear identification of TG vortices.



(a) (b)

Fig. 2. Magnitude of vorticity computed from (a) the instantaneous and (b) the fluctuation velocity fields. Blue and red colors correspond to the minimum and maximum values, respectively.

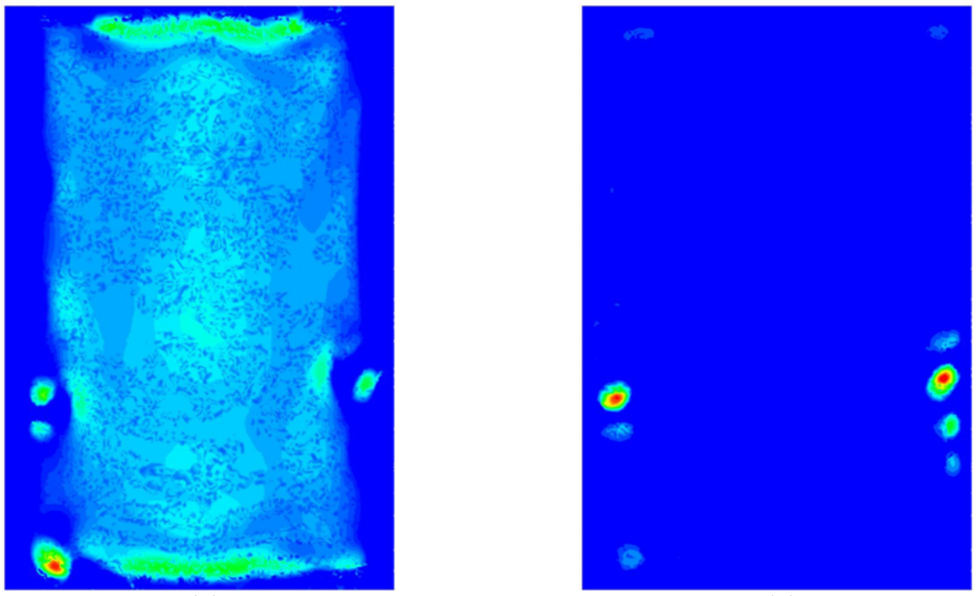
Figure 3(a) depicts the contours of the second invariant, Q, of the instantaneous velocity field. In this case the vortex cores are clearly identified. Moreover, in comparison with the previous method, no shear layers appeared. To remove the mean flow vortices, the fluctuation velocity field is applied instead of the instantaneous velocity field. In Fig. 3(b), the Taylor-Görtler vortices appear sharply as the dominant feature in the fluctuation field.



(a) (b)

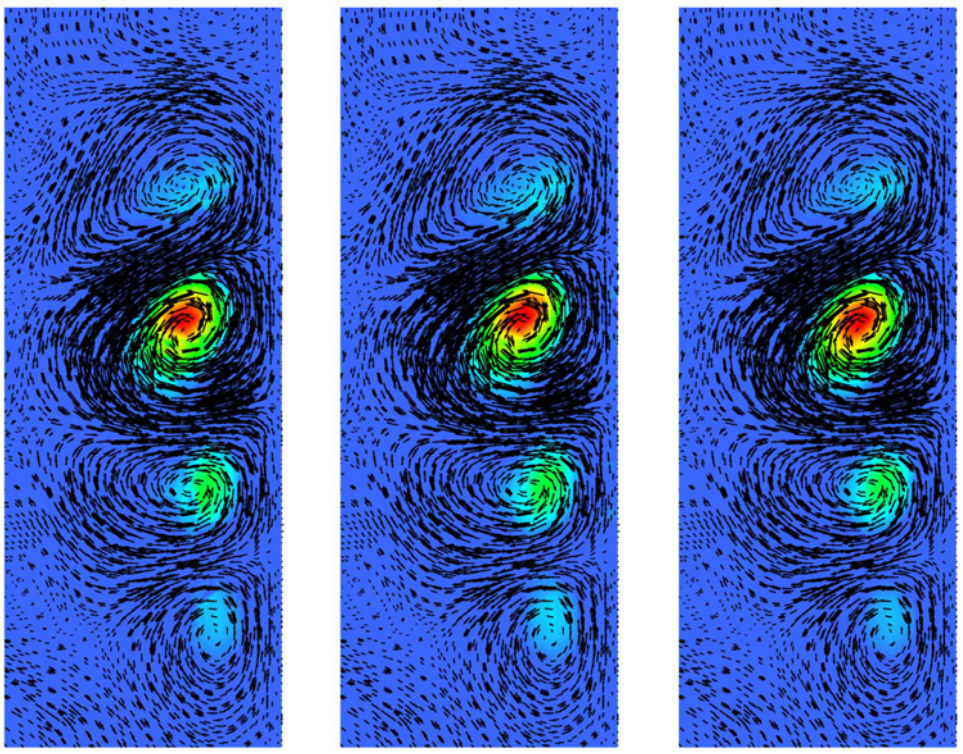
Fig. 3. Q contours of (a) the instantaneous and (b) the fluctuation velocity fields. Blue and red colors correspond to the minimum and maximum values, respectively.

Finally, the λ_2 method was applied. Figure 4(a) depicts the contours of λ_2 computed from the instantaneous velocity field. Both, the mean flow and TG vortices are well identified, but the vortex cores are distorted. Figure 4(b) shows the λ_2 contours in the fluctuation velocity field. The TG vortices are clearly visible, but slight distortions are present again.



(a) (b)

Fig. 4. λ_2 contours of (a) the instantaneous and (b) the fluctuation velocity fields. Red and blue colors correspond to the minimum and maximum values, respectively.



(a) (b) (c)

Fig. 5. Contours of (a) λ_2, (b) $\lambda_2 + \lambda_3$, and (c) $\lambda_1 + \lambda_2 + \lambda_3 = -2Q$. Vectors depict the fluctuation velocity.

The difference between the λ_2 and Q methods deserves further discussion. Both methods refer to the eigenvalues of $\mathbf{S}^2 + \mathbf{\Omega}^2$. Figures 5(a)-(c) depict the contours of λ_2, $\lambda_2 + \lambda_3$ and $\lambda_1 + \lambda_2 + \lambda_3$ in a region containing several TG vortices. For clarity the fluctuation velocity vectors are also shown in all three cases. Evidently, the λ_2 contours nicely fit to the vortex cores which are indicated by the velocity field (Fig. 5(a)). Adding the smallest eigenvalue, λ_3, is expected to emphasize stronger vortices. In the present case, however, this effect is only marginal (Fig. 5(b)). Including also the largest (possibly positive) eigenvalue, λ_1, could render weaker vortices invisible. As shown in Fig. 5(c), this phenomenon appears to be of minor importance, here. In fact, all three indicators are very close to each other. The conclusion is that the smallest and largest eigenvalues are of no significance for vortex identification in the given flow field. On the other hand, adding λ_1 and λ_3 or, equivalently, using Q as the indicator yields a smoother representation of vortices. Obviously, the sum of eigenvalues is less sensitive to numerical noise than λ_2 alone. This observation is important because in practice such noise occurs often as a consequence of numerical differentiation. The pressure-stabilized finite-element method used for discretization allows for very small but finite pressure oscillations. These and related oscillations in the velocity field are practically invisible but can be amplified when computing the first derivatives.

4. Three-Dimensional Visualization

The 2D investigations revealed that the Q criterion applied to the fluctuation field is well suited for identifying the instantaneous vortex structures in RMF-driven flow. Taking this into account, the next step is to apply this method to the three-dimensional flow field. Due to its sensitivity to numerical noise, the λ_2 criterion is not considered here.

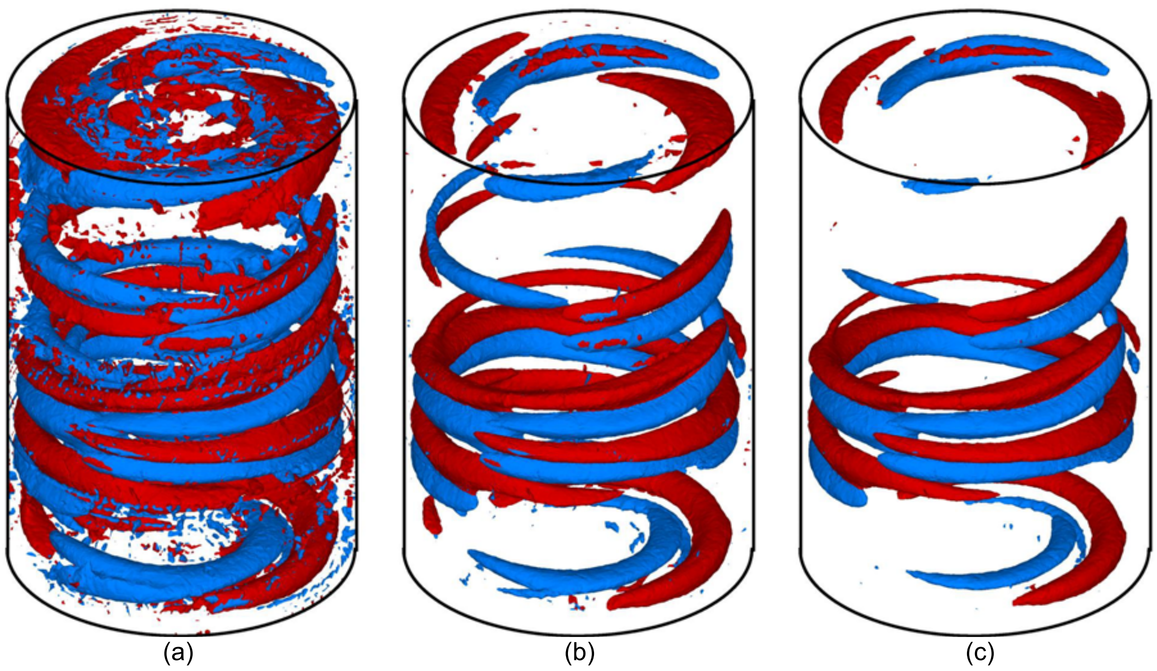


Fig. 6. Taylor-Görtler vortices identified using Q criterion applied to the fluctuation velocity field: (a) Q_0/Q_{max} = 0.00225, (b) Q_0/Q_{max} = 0.015 and (c) Q_0/Q_{max} = 0.035. Blue and red colors indicate the direction of rotation.

The vortex cores can be visualized using the iso-surface related to a properly chosen value Q_0. Figure 6 demonstrates the effect of different choices for Q_0. By definition, a vortex is identified with the region where $Q > 0$. In general, however, more instructive results are obtained choosing a greater threshold. However, the specification of an appropriate value for Q_0 is not simple task. Figure 6(a) depicts the case Q_0/Q_{max}= 0.0025. The vortex structures are well identified but affected by numerical noise. The choice Q_0/Q_{max} = 0.035 renders the weaker vortex structures invisible and only strong vortices are retained (Fig. 6(c)). As a consequence, a suitable value of Q_0 will be located between these two extremes. Figure 6(b) depicts the vortex structures visualized using Q_0/Q_{max} = 0.015. In this case the noise almost disappeared while smaller, yet relevant details are still identified.

From a practical point of view, the successful identification and visualization lends the way to a detailed investigation of the vortex structures. For instance, Fig. 5 reveals that the TG vortices preferably appear as pairs of counter-rotating vortex rolls. To indicate the orientation of rotation, the vortices are colored according to the sign of azimuthal vorticity. A red (blue) color corresponds to a positive (negative) sign and, hence, clockwise (anti-clockwise) rotation. Furthermore, a number of important features of TG vortices can be deduced. These include their spatial distribution, shape and dimensions. Moreover, by considering sequences of snapshots the temporal development can be studied as well.

5. Conclusion

Several techniques have been studied to explore the three-dimensional vortex structures in the unsteady flow driven by a rotating magnetic field. In particular we considered three different criteria for identifying the vortex core, namely the magnitude of vorticity, and the so-called Q and λ_2 criteria. The efficiency of these criteria was investigated at hand of a snapshot of the instantaneous velocity field provided by direct numerical simulation. In summary, only the Q criterion produced satisfactory results. The vorticity modulus failed to isolate the vortex cores due to its inability to avoid the appearance of shear layers. The λ_2 method was able to identify the vortices. However, the resulting vortex contours are significantly spoiled by numerical noise. As a consequence, only the Q method turned out to be appropriate for the present type of flow.

Apart from the criterion itself, it is important which representation of the flow is chosen for visualization. In the present case, the instantaneous velocity field is strongly influenced by the mean flow which is, in essence, dominated by large vortices, too. As a consequence, a significantly better visualization of the turbulent Taylor-Görtler vortices is achieved by using the fluctuation field only. In fact, this approach provided a first detailed insight into the three-dimensional vortex structure and dynamics of the turbulent flow induced by a RMF.

Acknowledgements

Financial support from German Deutsche Forschungsgemeinschaft in frame of the Collaborative Research Center SFB 609 is gratefully acknowledged.

References

Chong, M. S., Perry, A. E. and Cantwell, B. J., A general classification of three-dimensional flow field. Phys. Fluids A 2 (1990), 765.

Grants, I. and Gelfgat, Yu. M., Stability of swirling-recirculating flow due to rotating magnetic field, Magnetohydrodynamics, 34-1 (1998), 93-101.

Grants, I. and Gerbeth, G., Linear three-dimensional instability of a magnetically driven rotating flow, J. Fluid Mech., 463 (2002), 229-239.

Hunt, J. C. R., Wray, A. A. and Moin, P., Eddies stream, and convergence zones in turbulent flows, Center for Turbulence Research Report CTR-S88 (1988), 193.

Jeong, J. and Hussain F., On the identification of a vortex, J. Fluid Mech., 285 (1995), 69-94.

Kaiser, T. H. and Benz, K. W., Taylor vortex instabilities induced by a rotating magnetic field: A numerical approach, Physics of Fluids, 10 (1998) 5, 1104-1110.

Marty, Ph., Martin Witkowski, L., Trombetta, P. and Tomasino, T., On the stability of rotating MHD flows, Transfer Phenomena in Magnetohydrodynamic and Electroconducting Flows, (1999), 327-343, Kluwer Academic Publisher.

Metcalfe, R. W., Hussain, F., Menon, S. and Hyakwa, M., Coherent structures in a turbulent mixing layer: a comparison between numerical simulations and experiments. In Turbulent Shear Flows 5 (ed. F. Durst, B. E. Launder, J. L. Lumley,

F. W. Schmidt and J. H. Whitelaw), (1985), 110, Springer.
Stiller, J., Fraňa K., Grundmann, R., Fladrich, U. and Nagel, W. E, A Parallel PSPG Finite Element Method for Direct Simulation of Incompressible Flow, In: Euro-Par 2004, (2004), 726-733, Springer.

Author Profile

Karel Fraňa: He received his M.Sc. (Eng.) degree in 1999 from Technical University of Liberec in Czech Republic and his Ph.D. in 2004 from the same university. Nowadays he works at Institute of Aerospace Engineering at TU Dresden, Germany. His research interests focus on the numerical method (finite element method) and on the magnetically driven flow investigated by using direct numerical simulations.

Jörg Stiller: He is a lecturer (Dozent) for magnetofluiddynamics at the Institute for Aerospace Engineering at TU Dresden, Germany. He received his Ph.D. in 1994 from TU Dresden. His research activities focus on numerical methods, including adaptive finite element and spectral element techniques for incompressible and compressible flows.

Roger Grundmann: He studied turbo-engineering from 1964 to 1970 at Technical University of Berlin, Ph.D. in 1972 at TU Berlin. From 1972 until 1992 he was as a research scientist at DLR in Cologne and Göttingen and from 1985 to 1991 a professor at the Van Karman Institute for Fluid Dynamics in Brussels. Since 1993 he was a professor for thermofluiddynamics and applied aerodynamics at the TU Dresden. His research interests are fluid mechanics, computational fluid dynamics, aerodynamics and turbulence modeling.